Prerequisites

lecture

assume study of
- classical mechanics
- Special relativity
- Electromagnetism

at undergraduate level in physics

Main results:

$$\frac{d\underline{p}}{dt} = e\left(\underline{E} + \underline{v} \times \underline{B}\right)$$

where $\underline{p} = \gamma m \underline{v}$

m = invariant mass
$\underline{v}$ = 3-velocity
$\gamma = \dfrac{1}{\sqrt{1 - v^2/c^2}}$

+ Maxwell's equations (in free space with charges/currents)

$$\operatorname{div} \underline{E} = \frac{\rho(\underline{r}, t)}{\varepsilon_0} \quad \text{(gauss)}$$

ρ = charge density
$\underline{j}$ = current density.

differential form

$$\operatorname{div} \underline{B} = 0$$

$$\operatorname{curl} \underline{E} = -\frac{\partial \underline{B}}{\partial t} \quad \text{(Faraday)}$$

$$\operatorname{curl} \underline{B} = \mu_0 \underline{j}(\underline{r}, t) + \frac{1}{c^2}\frac{\partial \underline{E}}{\partial t} \quad \text{(Ampere)}$$

exercise: write these in integral form

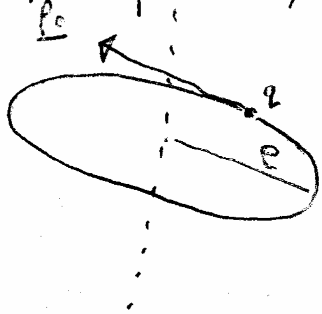
$$\iint \underline{E} \cdot d\underline{S} = \frac{1}{\varepsilon_0} \iiint \rho \, dV \qquad \iint \underline{B} \cdot d\underline{S} = 0$$

Goal: to store a collection ($\equiv$ beam) of particles for an extended period of time

⟶ create a uniform field B_0

⟶ store a single particle, charge q, momentum p_0

$\uparrow \underline{B} = B_0 \underline{\hat{y}}$



Q: why u[se] $\underline{B}$ for stor[age] and not $\underline{E}$

not a strict derivation!

Lorentz force law states $\underline{F} = q(\underline{E} + \underline{v} \times \underline{B})$

decompose $\underline{v}$ of particle into $V_\parallel$, $V_\perp$ to field

$V_\parallel$ component feels no force

$V_\perp$ component has $\theta = 90°$ to $\underline{B}$ $(\underline{V} \perp \underline{B})$

$$q(\underline{E} + \underline{V} \times \underline{B}) \rightarrow qVB_0$$

$$= \frac{\gamma m V^2}{\rho}$$

so we observe the particle moves in a circle

$\frac{dp}{dt} = q\,\underline{v} \times \underline{B}$

$\omega_0 p = qvB \Rightarrow \omega_0 = \frac{qvB}{\gamma m v}$

$\rho = \frac{v}{\omega_0} = \frac{v \cdot \gamma m v}{qvB} = \frac{p}{qB}$

$$\Rightarrow \frac{\gamma m V}{\rho} = q B_0 \Rightarrow \boxed{p_0 = q B_0 \rho}$$

define rigidity, $r = B_0 \rho = \frac{p_0}{q}$ (note, in terms of p_0, not E)

Units $r = \frac{p_0}{q} = B_0 \rho$ is in standard units

eg p_0 in $kg\,m\,s^{-1}$ etc

but what is the rigidity of a 500 GeV $\bar{e}$ beam?

$e = 1.602 \times 10^{-19}\,C$ $\quad c = 2.99 \times 10^{8}\,ms^{-1}$

So energy ÷ c = momentum $\Rightarrow \frac{1\,eV}{c} = 5.36 \times 10^{-28}$ kg ms^{-1}

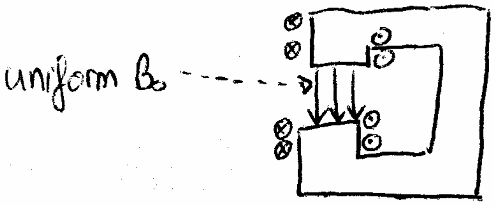
$\Rightarrow \frac{1\,GeV}{c} = 5.36 \times 10^{-19}$ kg ms^{-1}

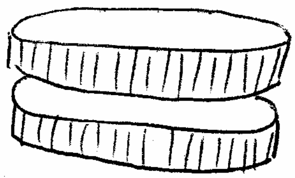
$\frac{5.36 \times 10^{-}}{q}$

$\Rightarrow B_0 \rho\,[T \cdot M] = 3.3\, P_0\,[GeV/c]$

(don't get lazy and drop the c's!!)

We've created a <u>storage ring</u> for our particle.
How do we realise the required magnetic field?





exercise

imagine a 20 TeV storage ring, with 17000 proton bunches, each with 1×10^{10} protons

1) What is the stored energy [in Joules] of this be

2) Circumference of ring is 83 km. If field is 6.6 T, what fraction of the ring is filled with dipo

can easily burn a hole in beampipe → 1) $20 \times 10^{12} \times 17000 \times 1 \times 10^{10} \times e^{-1}$

2) $B\rho = 66700\ T \cdot M \Rightarrow \rho = 10.1$ km of bendin required. $2\pi\rho/c = 76\%$

how do we make this magnet?

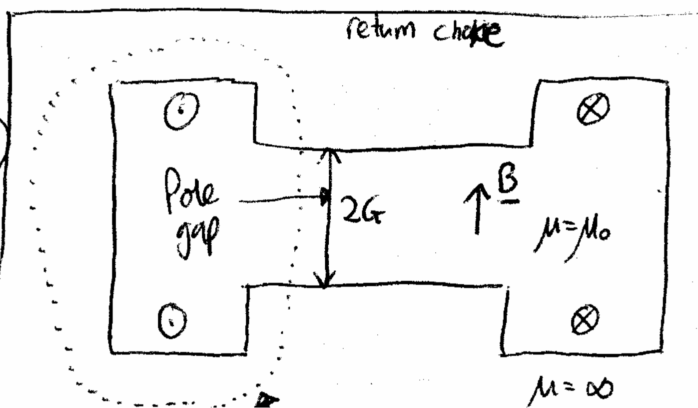
$\frac{1}{\mu} \text{curl}\, \underline{B} = \underline{J}$

$\Rightarrow \frac{1}{\mu} \oint \underline{B} \cdot \underline{dl} = I_{enc}$

$\mu_{iron} = \mu_r \mu_0$, $\mu_r = 10^4$

in the iron $\mu \sim \infty$

in the gap $\frac{B_0}{\mu_0} 2G$ $\int_{gap} \underline{dl}$



$$\Rightarrow B_0 = \frac{\mu_0 I_{tot}}{2G} \quad \text{or} \quad I_{tot}^{req} = \frac{B_0 2G}{\mu_0}$$

recall $1\,T = 10^4$ gauss $\quad \mu_0 = 4\pi \times 10^{-7}$

$$\Rightarrow I_{tot}^{req} = \frac{B_0[g] \times 10^{-4} \times 2 \times G[cm] \times 10^{-2}}{4\pi \times 10^{-7}}$$

$$= \frac{B_0[g]\, G[cm]}{0.2\,\pi}$$

So $B_0 = 1\,T = 10^4$ gauss $\quad G = 10\,cm \quad \Rightarrow \underline{I_{tot} = 150\,kA}$

<u>All well and good.</u> But Problem! what if $P_0 \rightarrow P_0 + \Delta$

$\Rightarrow$ then the field will be "wrong" for this moment

we only have the correct field for an <u>Ideal</u> partic

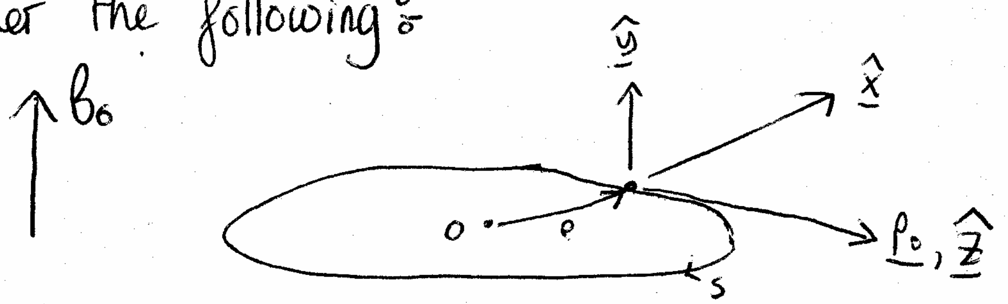
> " A Stable storage ring must store <u>non-ideal</u> particles, with slight deviations from the ideal condition i.e. accelerators must have a finite <u>acceptance</u> around the ideal conditions"

BASIC STABILITY CRITERIA

i.e. must store <u>beam</u> of particles with momentum spread + spatial/angular spread.

consider the following:

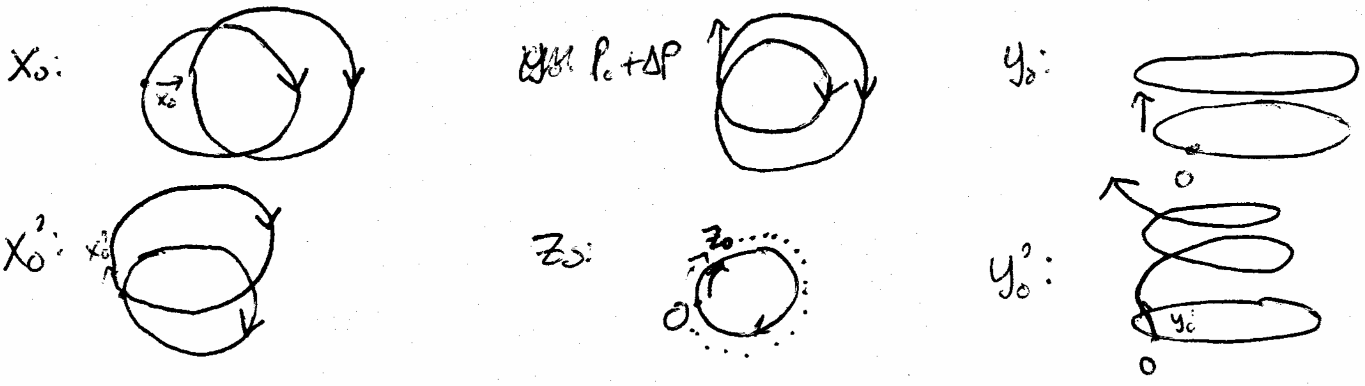
$\underline{B} = B_0 \underline{\hat{y}}$



what deviations must we consider in our stability princ

in 3D space we need 6D phase space

$\begin{pmatrix} x \\ x' \end{pmatrix} \quad \begin{pmatrix} x \\ x' \end{pmatrix} \longrightarrow 6$ possible deviations



So, we are stable in 5 dimensions, but slight deviations in y' will cause particles to leak out i.e. we are unstable, and a beam injected into this storage ring will eventually leak out

We make things stable with a clever trick.
→ We introduce a horizontal magnetic field component
introduce $B_x \propto -y \Rightarrow \underline{B} = B_0\hat{y} + Gy\hat{x}$ $(G<0)$

extra force if particle moves in z direction $\underline{v} = (0, 0, v_z)$

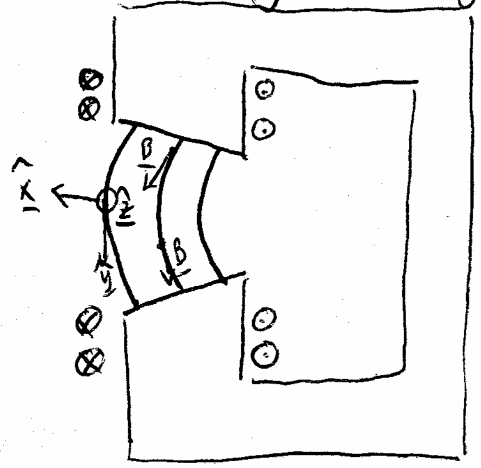
$\underline{f} = q\underline{v}\times\underline{B}$ $\quad q\underline{v} \times (Gy\hat{x}) \propto Gy\hat{y}$

particle is kicked towards $-\hat{y}$ (down) if $\hat{y} > 0$
→ particle is focused towards $y=0$ if $G<0$

$\Rightarrow$ $\boxed{\text{A Stable Storage ring}}$

we make this field by tilting the magnet pole face

pole gap is a function of x
$G \longrightarrow g(x)$



get
$\underline{B} = B_0\hat{y}$
and $-y\hat{x}$ bit

We have designed a "Combined function magnet"

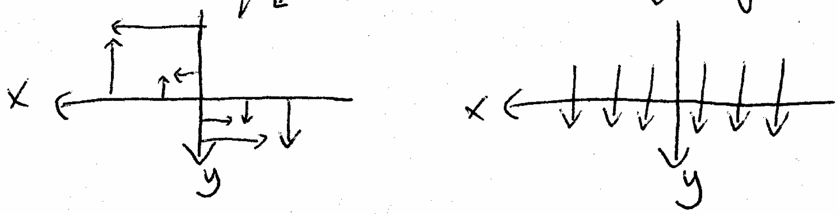
Recall the Maxwell equation in free space $\boxed{\nabla\times\underline{B}=0}$
This must be obeyed in the gap region of our magnet
here, $\underline{B} = B_0\hat{\underline{y}} + G y \hat{\underline{x}}$
taking the curl of this equation $\neq 0$ $\left(\begin{array}{c}\partial_i (B_0)=0\\ \partial_i(y\hat{\underline{x}})\neq 0\end{array}\right)$

can arrange for $\nabla\times\underline{B}=0$ by adding an component of B_y
s.t. $B_y \propto x$ now, $\underline{B} = B_0\hat{\underline{y}} + G(y\hat{\underline{x}} + x\hat{\underline{y}})$

now curl $\underline{B} = 0$ $\begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \partial x & \partial y & \partial z \\ Gy & Gx & 0 \end{vmatrix} = 0$

notice $B_x = Gy \Rightarrow G = \partial B_x / \partial y$ and $G = \partial B_y / B_x$.

So we have a quadrupole and a dipole field



G<0

So we have stable y motion, but extra $B_y = Gx$ term causes a new problem - for $G<0$ it defocuses in x

	x	y
$G<0$	defocusing	focusing
$G>0$	focusing	defocusing

<u>the field index</u>

let $g(x)$ be the pole face gap, Using Ampere's law as before, with $\mu=\infty$ in the iron

$$B_y(x) \propto \frac{1}{g(x)} \Rightarrow B_y(x) = \frac{B_0\, g(0)}{g(x)}$$

So $B_y(x) = B_0$ when $x=0$

Define the field index $\boxed{n = -\frac{\rho}{B_0}\frac{\partial B_y}{\partial x}}$

later in course, I'll show $0 < n < 1$

$$n = -\frac{\rho}{B_0}\frac{\partial}{\partial x}\left[\frac{B_0\, g(0)}{g(x)}\right] = -\rho\, g(0)\frac{\partial}{\partial x}\left[\frac{1}{g(x)}\right]$$

$$= -\rho\, g(0)\left(\frac{-g'(x)}{g^2(x)}\right)$$

Now assume x is small $\Rightarrow$ close to zero

$$\boxed{n = \frac{\rho}{g(0)}\, g'(0)}$$

also, we require $0 < n < 1$

$$\Rightarrow 0 < \frac{\rho g'}{g} < 1$$

$$\Rightarrow 0 < g' < \frac{g}{\rho}$$

g/ρ is very small $\Rightarrow$ tilt angle is very small

$\Rightarrow$ Beam dynamics are sensitive to magnet detail

$\Rightarrow$ we must be very careful in constructing magnets.

A combined function magnet works if G is small enough, as there is also some defocusing

G is weak $\Longrightarrow$ a weak focusing accelerator

We can also design a ring with separated function magnets i.e. pure dipole, pure quadrupole

BEAM $\longrightarrow$ [dipole] $\longrightarrow$ [quad] $\longrightarrow$ [dipole] $\longrightarrow$ [quad] $\longrightarrow$

A "Strong focusing" accelerator (more later)

(end of lecture)