# *Lecture 19: Huyghens Construction*

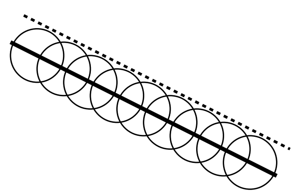
## 1. Huyghens - general

Huyghens (or Huygens, or Hugens) Construction (or principle) provides a way of answering the general problem of what happens when a particular initial wave moves through various regions of reflection, absorption, changing refractive index and so forth. It is a short and much easier (and more intuituve) alternative so solving the wave equation(s) in the different regions, and satisfying boundary conditions. It applies equally well in 2D and 3D (but 2D is discussed more as it's easier to draw.) It is often discussed in terms of light waves, but applies equally well to all other sorts of wave.

It says: you take a *wavefront* and consider each point on the wavefront as a source of waves. These spread out as little circles. From these you form the envelope, and that prescribes the next wavefront. Only the envelope need be considered, as in other regions the contributions from the different sources all cancel out. There may be a backward-going envelope but that is discarded as unphysical.

These two assumptions can be justified. Each source provides an excitation which is in fact an oscillation about zero, with zero integral. (This is obvious for water waves, as any water in a peak has to come from a trough. It follows for other waves by analogy.) The region in the middle receives contributions from several sources and they integrate over space to zero. The backward-going envelope ('back wave') receives contributions from this wave front and from the wavefront(s) at slightly earlier times, so they integrate over time to zero.

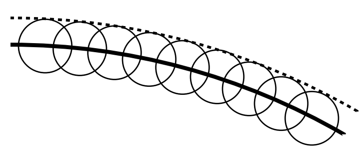
## 2. Plane wave



For a wavefront (heavy line) the circles are drawn and the new wavefront is the common tangent. So waves (the ray) proceeds normal to the direction of the wavefront.

## 3. Circular waves

For a circular (or spherical) wavefront, the envelope is a circle (or sphere) with greater radius. The wave expands.



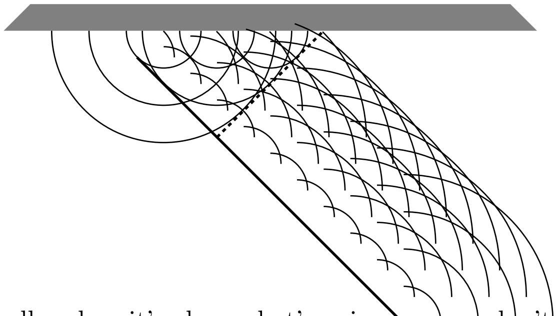
This is what happens when you throw a stone into a pond, and the ripples spread out in an ever-growing circle.

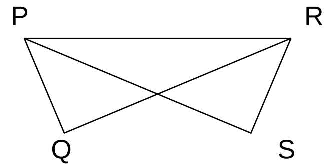
## 4. Reflection

In a ray of light (or other wave...) the wavefronts are normal to the ray, as the direction of motion is normal to the wavefront - see (1) above.

A little geometry shows that when you have a ray striking a surface, the angle between the ray and the normal-to-the-surface is equal to the angle between the wavefront and the surface itself. This is $\theta_i$ for the incident wave and $\theta_r$ for the reflected wave.

The wavefronts proceed towards the surface, with successive envelopes. As the edge of the wavefront hits the surface (it can go no further) a new envelope appears.





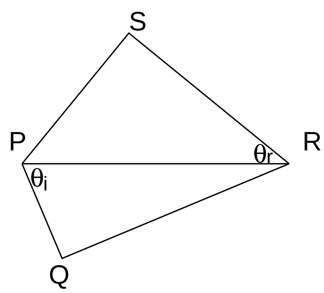

Actually when it's clear what's going on, you don't need to draw all the circles. It just takes 2 points to define a straight (2D) wavefront. In the figure, PQ is the wavefront just meeting the reflector at P. The wavefront at Q continues to meet the reflector at R, and meanwhile the excitation from P has generated a wavefront at S.

As the speed is constant, the distances QR and PS are equal. The angles PSR and PQR are both right angles, as the ray is normal to the wavefront. Pythagoras theorem then tells us that the sides are all the same and the triangles are congruent. This means that the angle QPR is equal to the angle PRS.

Now QPR is the angle between the wavefront and the mirror. It is also the angle between the incoming ray, which is normal to the incoming wavefront and the normal to the mirror. So it's the angle of incidence. Likewise PRS is the angle between the outgoing ray and the normal to the mirror. Huyghens' construction shows that the angle of incidence and the angle of reflection are equal. $\theta_i = \theta_r$

## 5. Refraction

Again we have $\theta_i$ and $\theta_r$ except that $\theta_r$ stands for the *refracted* wave. Suppose, as before, that PQ defines a wavefront and that this wavefront meets a medium-boundary at P. Again, the wave from Q proceeds to meet the boundary at R,with speed (say) $v$, but this time the wave from P proceeds inside the medium with speed (say) $v'$ to S. If the time taken is $t$ then the lengths are $vt$ and $v't$




Again, PQR and PSR are right angles. But the triangles are not congruent because the sides QR and PS are of different length. However they do have the same hypotenuse PR - call the length $h$. So $sin\theta_i = \frac{vt}{h}$ and $sin\theta_r = \frac{v't}{h}$. Eliminating the irrelevant $t$ and $h$ this gives *Snell's Law*: $\frac{sin\theta_i}{sin\theta_r} = \frac{v}{v'} = n$ where $n$ is the refractive index.

Snell's law is memorable except that it's easy to get it the wrong way up. For glass in air $v' < v$ then $\theta_r < \theta_i$ and the ray bends towards the normal as light goes from air to glass. From glass to air it bends away from the normal again. For example. If a light ray hits a glass block ($n = 1.3$) at 45 degrees, then $sin\theta_i = 0.7$, $sin\theta_r = 0.543$, and $\theta_r = 33$ degrees.

## 6. A Single slit:

If a plane wave strikes a screen which absorbs everything except at one point (a slit in 2D, a pinhole in 3D) then only one element need be considered. Plane waves become circular (or spherical) waves.